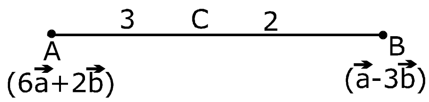
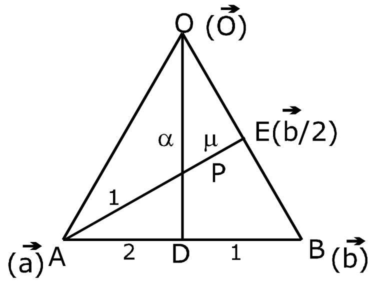
EXERCISE – III
HINTS & SOLUTIONS

Sol.1 $\vec{c} = \dfrac{3(\vec{a} - 3\vec{b}) + 2(6\vec{a} + 2\vec{b})}{5}$



$\vec{c} = 3\vec{a} - \vec{b}$

Sol.2 $P = \dfrac{\alpha\left(\dfrac{2\vec{b} + \vec{a}}{3}\right)}{\alpha + 1} = \dfrac{\dfrac{\vec{b}}{2} + \mu\vec{a}}{\mu + 1}$



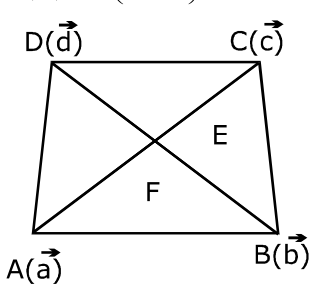
$\dfrac{2\alpha}{3(\alpha + 1)} = \dfrac{1}{2(\mu + 1)}$ ... (1)

$\dfrac{\alpha}{3(\alpha + 1)} = \dfrac{\mu}{(\mu + 1)}$ ...(2)

(1) – (2)

$2 = \dfrac{1}{2\mu} \Rightarrow \mu = 4 \Rightarrow \alpha = \dfrac{3}{2} \Rightarrow$ OP : PD = 3 : 2

Sol.3 $E\left(\dfrac{\vec{c}}{2}\right)$; $F\left(\dfrac{\vec{b} + \vec{d}}{2}\right)$



$\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD}$

$= \vec{b} + \vec{d} + (\vec{b} - \vec{c}) + (\vec{d} - \vec{c} = 2(\vec{b} + \vec{d}) - 2\vec{c}$

$\overrightarrow{EF} = \dfrac{\vec{b} + \vec{d}}{2} - \dfrac{\vec{c}}{2} = \dfrac{1}{2}[(\vec{b} + \vec{d}) - \vec{c}]$

so, $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$

Sol.4 $\vec{r} = (1, 2, 3) + \lambda \underbrace{(1, -1, 1)}_{\vec{d}}$

$\vec{r} = (1, 2, 3) + \mu \underbrace{(1, 1, -1)}_{\vec{d}}$

$\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{1 - 1 - 1}{\sqrt{3}\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$

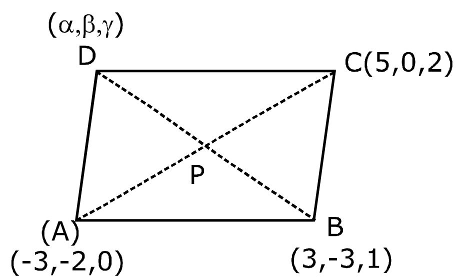
so acute angle bisector

$\vec{r} = (1, 2, 3) + t\,(\hat{a} - \hat{b})$

$= (1, 2, 3) + \dfrac{t}{\sqrt{3}}\,[(1, -1, 1) - (1, 1, -1)]$

$= (1, 2, 3) + \lambda(\hat{j} - \hat{k})$

Sol.5(i) Diagonal bisect each other



$\dfrac{3 + \alpha}{2} = \dfrac{5 - 3}{2}$

$\alpha = -1$

$-3 + \beta = -2 \Rightarrow \beta = 1$

$\gamma + 1 = 2 \Rightarrow \gamma = 1$

D(–1, 1, 1)

(ii) $\overrightarrow{AB} = (6, -1, 1)$ $\overrightarrow{AC} = (8, 2, 2)$

$|\overrightarrow{AB}| = \sqrt{38}$ $|\overrightarrow{AC}| = 6\sqrt{2}$

Reqd. vector $= \dfrac{(6, -1, 1)}{\sqrt{38}} \cdot 6\sqrt{2}$

$= \dfrac{6}{\sqrt{19}}(6, -1, 1)$

(iii) $\overrightarrow{AC} = (8, 2, 2)$ $\overrightarrow{BD} = (-4, 4, 0)$

$\cos\theta = \dfrac{\overrightarrow{AC}.\overrightarrow{BD}}{|\overrightarrow{AC}||\overrightarrow{BD}|} = -\dfrac{1}{2} \Rightarrow \theta = \dfrac{2\pi}{3}$

Sol.6(i) $|\vec{e}_1 - \vec{e}_2|^2 = 1$

$1 + 1 - 2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

(ii) $\left(\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2}\right)^2 = \left|\frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2}\right|^2$

$= \frac{a^2}{a^4} + \frac{b^2}{b^4} - \frac{2\vec{a}.\vec{b}}{a^2b^2} = \frac{1}{a^2} + \frac{1}{b^2} - 2\frac{\vec{a}.\vec{b}}{a^2b^2}$

$\left(\frac{\vec{a}-\vec{b}}{|\vec{a}|-|\vec{b}|}\right)^2 = \frac{1}{a^2b^2}(a^2 + b^2 - 2\vec{a}.\vec{b})$

$\frac{1}{a^2} + \frac{1}{b^2} - \frac{2\vec{a}.\vec{b}}{a^2b^2}$ LHS = RHS.

Sol.7 (i) $\vec{c} = \lambda(\vec{a}\times\vec{b})$, $\vec{a} = (2, 3, -1)$, $\vec{b} = (1, -2, 3)$

$\vec{a}\times\vec{b} = 7\hat{i} - 7\hat{j} - 7\hat{k}$

$\vec{c}\,.(2, -1, 1) = 6 = 0$ $\vec{c} = \lambda(7, -7, -7)$

$\mu(2 + 1 - 1) + 6 = 0$ $= \mu(1, -1, -1)$

$\mu = -3 \Rightarrow \vec{c} = (-3, 3, 3)$

(ii) $|\vec{a}| = 10$; $|\vec{b}| = 10$ $\vec{a}.\vec{b} = 12$

$\vec{a}.\vec{b} = 12$

$|a|\ |b| \cos\theta = 12 \Rightarrow \cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}| \sin\theta = 10 \times 2 \times \frac{4}{5} = 16$

Sol.8 Shortest distance between two lines

$$d = \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$$

$\vec{a}_1 = (4, -1, 0)$ $\vec{a}_2 = (1, -1, 2)$

$\vec{b}_1 = (1, 2, -3)$ $\vec{b}_2 = (2, 4, -5)$

$\vec{a}_2 - \vec{a}_1 = (-3, 0, 2)$

$\vec{b}_1 \times \vec{b}_2 = (2, -1, 0)$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = -6$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{4+1+0} = \sqrt{5}$

Shortest distance $d = \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$

Sol.9 (i) $(\hat{m} \wedge \hat{n}) = \hat{p} \wedge (\hat{m}\times\hat{n}) = \alpha$

$[\hat{n}\ \ \hat{p}\ \ \hat{m}] = [\hat{p}\ \ \hat{m}\ \ \hat{n}] = \hat{p}\ .\ (\hat{m}\times\hat{n})$

$= [\hat{n}\ \ \hat{p}\ \ \hat{m}]\ .\ (\hat{m}\times\hat{n}) = |p| \cos\alpha\ (\hat{m}\times\hat{n})$

$= |p| \cos\alpha\ |m|\ |n| \sin a = \cos\alpha \sin\alpha$

(ii) $|[\vec{a}\ \ \vec{b}\ \ \vec{c}]| = |\vec{a}.(\vec{b}\times\vec{c})|$

$= \big||\vec{a}|\,|\vec{b}\times\vec{c}|\big| = |a|\ |b|\ |c| \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Sol.10 $\vec{a} = (1, 2, 3)$ $\vec{b} = (2, -1, 1)$

$\vec{c} = (3, 2, 1)$ $\vec{d} = (3, -1, -2)$

(i) $\vec{a}\times(\vec{b}\times\vec{c}) = p\vec{a} + q\vec{b} + r\vec{c}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$10\vec{b} - 3\vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$

p = 0, q = 10, r = –3

(ii) $(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c}).\vec{d}$ $[(\vec{a}\times\vec{b}) = p]$

$\vec{p}\times(\vec{a}\times\vec{c}).\vec{d}$

$[(\vec{p}.\vec{c})\vec{a} - (\vec{p}.\vec{a})\vec{c}]\,.\vec{d}$

$[\vec{a}\ \vec{b}\ \vec{c}](\vec{a}.\vec{d}) = (20)\,(-5) = -100$

Sol.11 $\vec{x} + \frac{1}{p^2}(\vec{p}\,.\,\vec{x})\,\vec{p} = \vec{q}$ and $\vec{p}\,.\,\vec{x} = \frac{1}{2}(\vec{p}\,.\,\vec{q})$

$\vec{x} + \frac{(\vec{p}\,.\,\vec{q})\vec{p}}{2p^2} = \vec{q} \Rightarrow \vec{x} = \vec{q} - \frac{(\vec{p}\,.\,\vec{q})\vec{p}}{2p^2}$

Sol.12(i) $\vec{a} = (1, -2, 3)$ $\vec{b} = (3, -6, 9)$

$\vec{b} = 3(1, -2, 3)$

$\vec{b} = 3\vec{a}$ Linearly dependent

(ii) $\vec{a} = (-2, 0, -4)$ $\quad \vec{b} = (1, -2, -1)$

$\vec{c} = (1, -4, 3) = 0$

$x\vec{a} + y\vec{b} + z\vec{c} = 0$

$x(-2, 0, -4) + y(1, -2, -1) + z(1, -4, 3) = 0$

$\Rightarrow x = 0, y = 0, z = 0$ Linear Independent

Sol.13 $\vec{x} = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{y} = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$; $\vec{z} = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$\vec{a}, \vec{b}, \vec{c}$ are non-coplanar $\Rightarrow$ $[\vec{a}\,\vec{b}\,\vec{c}] \neq 0$

$[\vec{x}\ \vec{y}\ \vec{z}] = \vec{x}.(\vec{y}\times\vec{z})$

$= \dfrac{1}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^3}\ [(\vec{b}\times\vec{c}).\{\underbrace{(\vec{c}\times\vec{a})}_{p}\times(\vec{a}\times\vec{b})\}]$

$= \dfrac{1}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^3}\ [(\vec{b}\times\vec{c}).\{(\vec{p}.\vec{b})\vec{a} - (\vec{p}.\,\vec{a})\vec{b})\}]$

$= \dfrac{1}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^3}\ [(\vec{b}\times\vec{c}).[\vec{a}\,\vec{b}\ \ \vec{c}]\,\vec{a}]$

$= \dfrac{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^3} = \dfrac{1}{[\vec{a}\ \ \vec{b}\ \ \vec{c}]} \neq 0$

$\vec{x}.(\vec{a}+\vec{b}) + \vec{y}.(\vec{b}+\vec{c}) + \vec{z}.(\vec{c}+\vec{a})$

$\vec{x}.(\vec{a}+\vec{b}) = \dfrac{(\vec{b}\times\vec{c})}{[\vec{a}\,\vec{b}\,\vec{c}]}.(\vec{a}+\vec{b})$

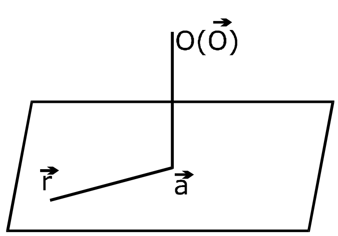
$= \dfrac{[\vec{b}\,\vec{c}\,\vec{a}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = 1$ ... (1)

Similarly $\vec{y}.(\vec{b}+\vec{c}) = 1$ ... (2)

$\vec{z}.(\vec{c}+\vec{a}) = 1$ ... (3)

So sum of (1) (2) (3) = 3

Sol.14



$\vec{a} = (4, -2, -5)$

$(\vec{r}-\vec{a}).\vec{a} = 0$

$\vec{r} - \vec{a} = |a|^2$

$\vec{r}.(4, -2, -5) = 16 + 4 + 25$

$\vec{r}.(4, -2, -5) = 45$

Sol.15 $\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) = 5$

$\vec{r}.(6\hat{i} - 9\hat{j} + 18\hat{k}) = -20$

$\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) = \dfrac{-20}{3}$

distance $= \dfrac{5 + \dfrac{20}{3}}{\sqrt{4+9+36}} = \dfrac{35}{3\times 7} = \dfrac{5}{3}$ units.

Sol.16(i) $\dfrac{1}{2}|\vec{a} - \vec{b}|$

$= \dfrac{1}{2}\sqrt{|a|^2 + |b|^2 - 2|a||b|\cos\theta}$

$= \dfrac{1}{2}\sqrt{1 + 1 - 2\cos\theta}$

$= \dfrac{1}{2}\sqrt{2(1 - \cos\theta)}$

$= \dfrac{1}{2}\sqrt{2.2\sin^2\dfrac{\theta}{2}} = \sin\dfrac{\theta}{2}$

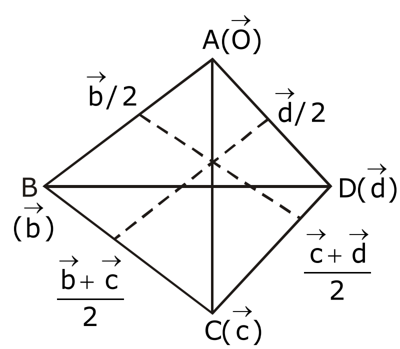
(ii) $\dfrac{1}{2}\ |\vec{a} + \vec{b}| = \dfrac{1}{2}\sqrt{1 + 1 + 2\cos\theta} = \cos\dfrac{\theta}{2}$

Sol.17

Given that

$\overrightarrow{AD}\,.\,\overrightarrow{BC} = 0$

$\vec{d}\,.\,(\vec{c} - \vec{b}) = 0$



$\Rightarrow \quad \vec{c}\cdot\vec{d} = \vec{b}\cdot\vec{d}$ ....(i)

& $\quad \overrightarrow{AC}\cdot\overrightarrow{BD} = 0$

$\Rightarrow \vec{c}\cdot(\vec{d}-\vec{b}) = 0 \Rightarrow \vec{c}\cdot\vec{d} = \vec{b}\cdot\vec{c}$ ....(ii)

Now $\overrightarrow{AB}\cdot\overrightarrow{CD} = \vec{b}\cdot(\vec{d}-\vec{c})$

$= \vec{b}\cdot\vec{d} - \vec{b}\cdot\vec{c} = 0$

Hence $\overrightarrow{AB} \perp \overrightarrow{CD}$

$|\overrightarrow{AD}|^2 + |\overrightarrow{BC}|^2 = d^2 + b^2 + c^2 - 2(\vec{b}\cdot\vec{c})$

$|\overrightarrow{AC}|^2 + |\overrightarrow{BD}|^2 = c^2 + b^2 + d^2 - 2(\vec{b}\cdot\vec{d})$

$= c^2 + b^2 + d^2 - 2(\vec{b}\cdot\vec{c})$

$|\overrightarrow{AB}|^2 + |\overrightarrow{CD}|^2 = b^2 + c^2 + d^2 - 2(\vec{c}\cdot\vec{d})$

$= b^2 + c^2 + d^2 - 2(\vec{b}\cdot\vec{c})$

Hence Proved.

Sol.18(i) $\vec{A} = (2, 0, 1)$ $\quad \vec{B} = (1, 1, 1)$

$\vec{C} = (4, -3, 7)$

$\vec{R}\cdot\vec{A} = 0$

$\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$

$\vec{A}\times(\vec{R}\times\vec{B}) = \vec{A}\times(\vec{C}\times\vec{B})$

$(\vec{A}\cdot\vec{B})\vec{R} - (\vec{A}\cdot\vec{R})\vec{B} = (\vec{A}\cdot\vec{B})\vec{C} - (\vec{A}\cdot\vec{C})\vec{B}$

$(\vec{A}\cdot\vec{B})\vec{R} = (\vec{A}\cdot\vec{B})\vec{C} - (\vec{A}\cdot\vec{C})\vec{B}$

$$\vec{R} = \vec{C} - \frac{\vec{A}\cdot\vec{C}}{\vec{A}\cdot\vec{B}}\vec{B} = \vec{C} - \frac{(8+7)}{(2+1)}\vec{B} = \vec{C} - \frac{5}{3}\vec{B}$$

$= \vec{C} - 5\vec{B} = (4, -3, 7) - 5(1, 1, 1)$

$\vec{R} = (-1, -8, -2)$

(ii) Let $\vec{V} = (\alpha, \beta, \gamma)$

$\vec{V}\cdot(\vec{a}\times\vec{b}) = 0$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = -3\hat{i} - 3\hat{j} - 3\hat{k}$$

$-3\alpha - 3\beta - 3\gamma = 0$ ... (1)

$-2\alpha + 3\beta - \gamma = 0$ ... (2)

$|\vec{V}|\cos\theta = 6\sqrt{3}$

$$|\vec{V}|\,\frac{\vec{V}\cdot\vec{C}}{|\vec{V}||\vec{C}|} = 6\sqrt{3}$$

$$\frac{\vec{V}\cdot\vec{C}}{|\vec{C}|} = 6\sqrt{3}$$

$$\frac{\alpha-\beta+\gamma}{\sqrt{3}} = 6\sqrt{3}$$

$\alpha - \beta + \gamma = 18$ ...(3)

By solving (1) (2) (3)

$\alpha = 0, \beta = -9, \gamma = 9$

$\vec{V} = 9(-\hat{j} + \hat{k})$

Sol.19

Let circumcentre $(\vec{O}) = \vec{O}$

$$G = \frac{H + 2O}{3} \Rightarrow H = 3G$$

$$\vec{G} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

$\vec{H} = \vec{a} + \vec{b} + \vec{c}$

$|\overrightarrow{OH}| = |\vec{a}+\vec{b}+\vec{c}|$

$= \sqrt{a^2 + b^2 + c^2 + 2\Sigma\,\vec{a}\cdot\vec{b}}$

$= \sqrt{3R^2 + 2R^2\Sigma\cos 2A}$

$= \sqrt{3R^2 + 2R^2(-1 - 4\cos A\cos B\cos C)}$

$= R\sqrt{1 - 8\cos A\cos B\cos C}$

Sol.20 $\vec{r}\cdot(3\hat{i} - \hat{j} + \hat{k}) = 1$ and $\vec{r}\cdot(\hat{i} + 4\hat{j} - 2\hat{k}) = 2$

$$\vec{p} = \overrightarrow{n_1}\times\overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 4 & -2 \end{vmatrix} = -2\hat{i} + 7\hat{j} + 13\hat{k}$$

Equation of planes

$3x - y + z = 1$ and $x + 4y - 2z = 2$

put $z = 0$ $\quad 3x - y = 1 \quad x = \frac{6}{13}\,;\; y = \frac{5}{13}$

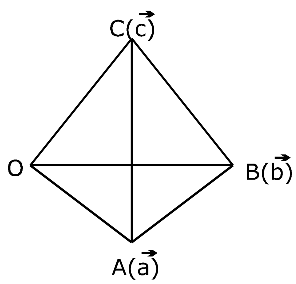
$x + 4y = 2$

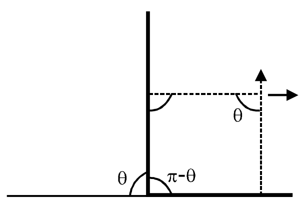
Equation of line will be

$$\frac{x - \frac{6}{13}}{-2} = \frac{y - \frac{5}{13}}{7} = \frac{z - 0}{13}$$

or $\vec{r} = \left(\frac{6}{13}, \frac{5}{13}, 0\right) + \lambda\,(-2, 7, 13)$

Sol.21





(a) a= b = c

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = 60^\circ$

Angle between planes = angle between their normals

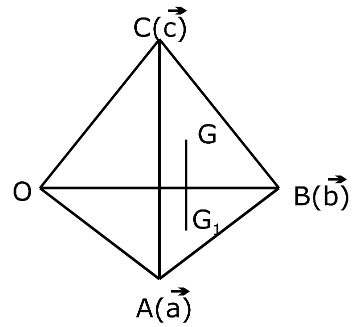
$$\cos\theta = \left|\frac{\overrightarrow{n_1}\,.\,\overrightarrow{n_2}}{|\overrightarrow{n_1}|\,|\overrightarrow{n_2}|}\right| = \left|\frac{(\vec{a}\times\vec{c}).(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{c}|\,|\vec{a}\times\vec{b}|}\right|$$

$$= \frac{((\vec{a}\times\vec{c})\times\vec{a}).\vec{b}}{a^2\sin 60^\circ\; a^2\sin 60^\circ} = \left|\frac{(a^2\vec{c} - (\vec{c}.\vec{a})\vec{a}).\vec{b}}{\frac{3}{4}a^4}\right|$$

$$= \frac{a^2(\vec{b}\,.\,\vec{c}) - (\vec{c}\,.\,\vec{a})(\vec{a}\,.\,\vec{b})}{\frac{3}{4}a^4}$$

$$\cos\theta = \frac{\frac{a^4}{2} - \frac{a^4}{4}}{\frac{3}{4}a^4} = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\frac{1}{3}$$

(b) Regular tetrahedron



so a = b =c = k

and $\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = 60^\circ$

$$\vec{G} = \left(\frac{\vec{a} + \vec{b} + \vec{c}}{4}\right)$$

Circumradius = Distance of G from any vertex

$= |\overrightarrow{OG}|$

$$= \frac{\sqrt{a^2 + b^2 + c^2 + 2\vec{a}\,.\,\vec{b} + 2\vec{b}\,.\,\vec{c} + 2\vec{c}\,.\,\vec{a}}}{4}$$

$$= \frac{\sqrt{3a^2 + 6a^2\cos 60^\circ}}{4} = \frac{a\sqrt{6}}{4} = \frac{k\sqrt{6}}{4}$$

Inradius $|GG_1|$

[Here $G \to$ Centroid of tetrahedron

$G_1 \to$ Centroid of face OAB]

$$= \left|\frac{\vec{a} + \vec{b}}{3} - \frac{\vec{a} + \vec{b} + \vec{c}}{4}\right| = \left|\frac{\vec{a} + \vec{b} - 3\vec{c}}{12}\right|$$

$$\frac{a\sqrt{6}}{12} = \frac{a}{2\sqrt{6}} = \frac{k}{2\sqrt{6}}$$

Sol.22 Let Point of intersection is P.

equation of plane CDE

$-3(x + 4) + 2\,(y - 0) - 11\,(z - 4) = 0$ ...(i)

Let any point on line AB is

P $(1 + \lambda,\, 2 - \lambda,\, 1 + \lambda)$

P will lie on plane ....(i)

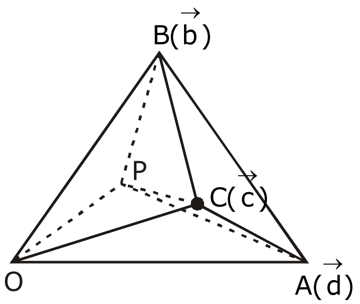
so $\lambda = 11/8$

so P(19/8, 11/8, 19/8)

Sol.23

$|\overrightarrow{PA}| = |\overrightarrow{PB}| = |\overrightarrow{PC}|$

$= |-\vec{p}| = |\vec{r}|$



$(\vec{a} - \vec{r}) \cdot (\vec{a} - \vec{r}) = r^2$

$(\vec{a} - \vec{r}) \cdot (\vec{a} \cdot \vec{r}) = r^2$

$\Rightarrow a^2 = 2\,\vec{a} \cdot \vec{r}\ ;\ b^2 = 2\,\vec{b} \cdot \vec{r}\ ;\ C^2 = 2\,\vec{c} \cdot \vec{r}$

Since $(\vec{b} \times \vec{c})$, $(\vec{c} \times \vec{a})$ and $(\vec{a} \times \vec{b})$ are non-coplanar.

$\vec{r} = x(\vec{b} \times \vec{c}) + y(\vec{c} \times \vec{a}) + z(\vec{a} \times \vec{b})$

$\vec{a} \cdot \vec{r} = x[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow x = \dfrac{a^2}{2[\vec{a}\ \vec{b}\ \vec{c}]}$

$y = \dfrac{b^2}{2[\vec{a}\ \vec{b}\ \vec{c}]}\ ;\ z = \dfrac{c^2}{2[\vec{a}\ \vec{b}\ \vec{c}]}$

$$\vec{r} = \frac{a^2(\vec{b} \times \vec{c}) + b^2(\vec{c} \times \vec{a}) + c^2(\vec{a} \times \vec{b})}{2[\vec{a}\ \vec{b}\ \vec{c}]}$$

Sol.24 (i) Let

$\vec{A} = (4, 8, 12)$

$\vec{B} = (2, 4, 6)$; $\vec{C} = (3, 5, 4)$

$\vec{D} = (5, 8, 5)$

$\overrightarrow{AB} = (-2, -4, -6)$; $\overrightarrow{AC} = (-1, -3, -8)$

$\overrightarrow{AD} = (1, 0, -7)$

$[\overrightarrow{AB}\ \ \overrightarrow{AC}\ \ \overrightarrow{AD}] = 0$

$\Rightarrow$ Hence points are coplanar.

(ii) Same as part (i)

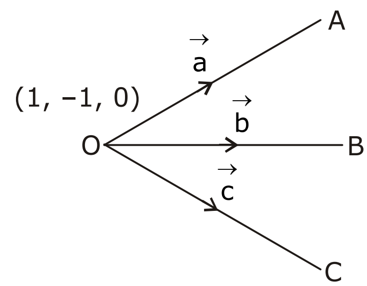
Sol.25 Vector normal to plane ABC is $8\hat{i} - 4\hat{j} + 3\hat{k}$

Vector normal to plane ADC is $-2\hat{i} + \hat{j} + 6\hat{k}$

So acute angle between plane ABC & ADC

can $\theta = \dfrac{2}{\sqrt{89}\sqrt{41}}$

Sol.26



$\vec{A} = (5, -3, 1)$

$\vec{B} = (3, -2, -1)$

$\vec{C} = (3, -1, 1)$

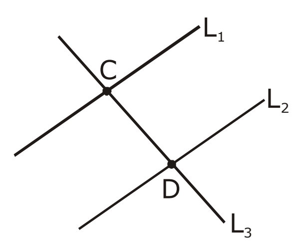
$\overrightarrow{AB} \times \overrightarrow{AC} = 4\hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{r} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = \vec{A} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$

$\vec{r} \cdot (2\hat{i} + 2\hat{j} - \hat{k}) = 3$

Sol.27 $L_1 : \vec{r} = (7, 6, 2) + \lambda(3, 2, 4)$

$L_2 : \vec{r} = (5, 3, 4) + \mu(2, 1, 3)$



$C(7 + 3\lambda, 6 + 2\lambda, 2 + 4\lambda)$
$D(5 + 2\mu, 3 + \mu, 4 + 3\mu)$

$\overrightarrow{CD} = (2\mu - 3\lambda - 2, \mu - 2\lambda - 3, 3\mu - 4\lambda + 2)$

Now

$$\frac{2\mu - 3\lambda - 2}{2} = \frac{\mu - 2\lambda - 3}{-2} = \frac{3\mu - 4\lambda + 2}{-1}$$

Get λ & μ

and Hence find distance $|\overrightarrow{CD}|$.